Math Virtual Learning

Precalculus with Trigonometry

April 30, 2020

Precalculus with Trigonometry
Lesson: April 30th, 2020

Objective/Learning Target:
Students will be able to find exact values of a trigonometric expression using the double-angle formulas.

Let’s Get Started:

Watch the video below to see how the double angle formulas are developed from the sum formulas in the previous two lessons.

Watch Video: Derivation of the double angle identities

Watch from the beginning for good introduction to the double angle formulas for sine, cosine, and tangent.

*** Please note that cosine has more than 1 formula.

Double Angle Identities

$$\sin 2x = 2\sin x\cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 2\cos^2 x - 1$$

$$= 1 - 2\sin^2 x$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

Finding Exact Values

Now that you've seen where the formulas come from, how can they be used?

Watch the video below to see how these formulas can be used to find the exact values of certain angles. You only have to watch till **3:15**, however I would suggest watching the whole video to get a good idea of how we will be using these formulas in the next couple days.

Video: How to Use Double Angle Formulas

Example 1: Evaluating Functions Involving Double Angles

Use the following to find $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$.

$$\cos\theta = \frac{5}{13}, \qquad \frac{3\pi}{2} < \theta < 2\pi$$

Solution

From Figure 5.15, you can see that $\sin\theta = y/r = -12/13$. Consequently, using each of the double-angle formulas, you can write

$$\sin 2\theta = 2\sin\theta\cos\theta = 2\left(-\frac{12}{13}\right)\left(\frac{5}{13}\right) = -\frac{120}{169}$$

$$\cos 2\theta = 2\cos^2\theta - 1 = 2\left(\frac{25}{169}\right) - 1 = -\frac{119}{169}$$

$$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{120}{119}.$$

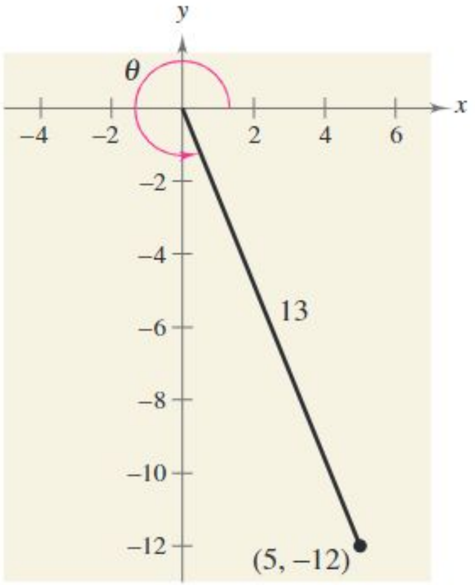
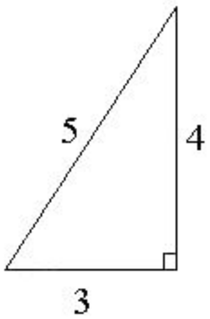


FIGURE 5.15

Example 2: Without finding x, find the exact value of $\sin 2x$ if $\cos x = \frac{4}{5}$ (in Quadrant I).

Solution: We recognise that we need to use the 3-4-5 triangle (because of the 4 and 5 in the question).



We can use our formula for the sine of a double angle to find the required value:

$$\sin 2x = 2 \sin x \cos x$$

$$= 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)$$

$$= \frac{24}{25}$$

Example 3:

Find cos 60° by using the functions of 30°.

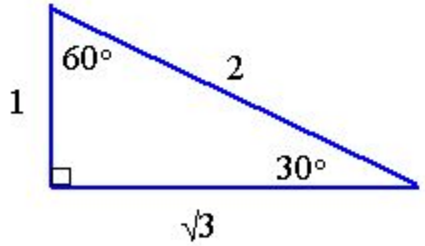
Solution:

We can only use the sine and cosine functions of 30^o, so we need to start with $60^o = 2 \times 30^o$.

We will use the result

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha,$$

and the 30-60 triangle:



Now we proceed to find the exact value of cos60° using the ratios of 30°:

$$\begin{aligned}\cos 60^o &= \cos\left(2 \times 30^o\right)\\ &= \cos^2(30^o) - \sin^2(30^o)\\ &= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2\\ &= \frac{3}{4} - \frac{1}{4}\\ &= \frac{2}{4}\\ &= \frac{1}{2}\end{aligned}$$

These exercises are really here for practice on the double angle formula. Of course, we could have found the value of cos60° directly from the triangle.

Example 4: Find the exact value of $\cos 2x$ if $\sin x = -\frac{12}{13}$ (in Quadrant III).

Solution:

Using the following form of the cosine of a double angle formula, $\cos 2\alpha = 1 - 2\sin^2 \alpha$, we have:

$$\cos 2x = 1 - 2\sin^2 x$$

$$= 1 - 2\left(\frac{-12}{13}\right)^2$$

$$= 1 - 2\left(\frac{144}{169}\right)$$

$$= \frac{169 - 288}{169}$$

$$= \frac{-119}{169}$$

Notice that we didn't find the value of x using calculator first, and then find the required value. If we had done that, we would not have found the **exact** value, and we would have missed the pleasure of seeing the double angle formula in action :-)

Practice

On a separate piece of paper, use the Double-Angle Identities to determine the exact values of sin(2x), cos(2x), and tan(2x) for each of the following situations.

1. $\sin x = \frac{3}{5} \quad and \quad 0 < x < \frac{\pi}{2}$

2. $\tan x = \frac{-7}{24} \quad and \quad \frac{\pi}{2} < x < \pi$

3. $\cos x = \frac{-3}{7} \quad and \quad \pi < x < \frac{3\pi}{2}$

Practice - **ANSWERS**

On a separate piece of paper, use the Double-Angle Identities to determine the exact values of sin(2x), cos(2x), and tan(2x) for each of the following situations.

Watch the following video for a walkthrough for each of these problems.

Video: Double Angle Identities & Formulas - Exact Value of Sin(2x), Cos(2x), Tan(2x)

Answers start at:

1. 1:01
2. 5:29
3. 8:04

1. $\sin x = \frac{3}{5}$ *and* $0 < x < \frac{\pi}{2}$

2. $\tan x = \frac{-7}{24}$ *and* $\frac{\pi}{2} < x < \pi$

3. $\cos x = \frac{-3}{7}$ *and* $\pi < x < \frac{3\pi}{2}$

Additional Practice and Resources:

Additional Resource Videos:

Double Angle Identities & Formulas of Sin, Cos & Tan - Trigonometry

Trigonometry - Finding Exact Value Using Double Angle Identities

Additional Practice:

Double & Half Angle Identities - KUTA (with solutions)

Do Problems 1-4, 10, 12, 13, 16, 17, 22, and 23

Double & Half Angle Formulas Practice (with solutions)

Do Problems 1-6